General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MPC3

Unit Pure Core 3

Monday 13 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

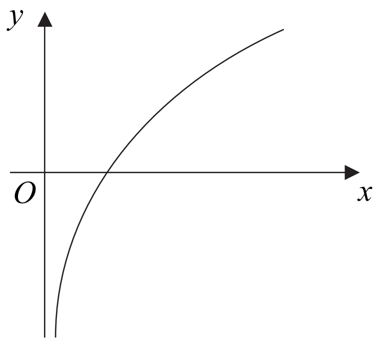
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The diagram shows the curve with equation $y = \ln(6x)$.



(a) State the x-coordinate of the point of intersection of the curve with the x-axis. *(1 mark)*

(b) Find $\dfrac{dy}{dx}$. *(2 marks)*

(c) Use Simpson's rule with 6 strips (7 ordinates) to find an estimate for $\displaystyle\int_1^7 \ln(6x)\,dx$, giving your answer to three significant figures. *(4 marks)*

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2 (a) (i) Find $\dfrac{dy}{dx}$ when $y = xe^{2x}$. *(3 marks)*

(ii) Find an equation of the tangent to the curve $y = xe^{2x}$ at the point $(1,\ e^2)$. *(2 marks)*

(b) Given that $y = \dfrac{2\sin 3x}{1 + \cos 3x}$, use the quotient rule to show that

$$\frac{dy}{dx} = \frac{k}{1 + \cos 3x}$$

where k is an integer. *(4 marks)*

3 The curve $y = \cos^{-1}(2x - 1)$ intersects the curve $y = e^x$ at a single point where $x = \alpha$.

(a) Show that α lies between 0.4 and 0.5. *(2 marks)*

(b) Show that the equation $\cos^{-1}(2x - 1) = e^x$ can be written as $x = \frac{1}{2} + \frac{1}{2}\cos(e^x)$. *(1 mark)*

(c) Use the iteration $x_{n+1} = \frac{1}{2} + \frac{1}{2}\cos(e^{x_n})$ with $x_1 = 0.4$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

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4 (a) (i) Solve the equation $\operatorname{cosec}\theta = -4$ for $0° < \theta < 360°$, giving your answers to the nearest 0.1°. *(2 marks)*

(ii) Solve the equation

$$2\cot^2(2x + 30°) = 2 - 7\operatorname{cosec}(2x + 30°)$$

for $0° < x < 180°$, giving your answers to the nearest 0.1°. *(6 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \operatorname{cosec} x$ onto the graph of $y = \operatorname{cosec}(2x + 30°)$. *(4 marks)*

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5 The functions f and g are defined with their respective domains by

$$f(x) = x^2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x + 1} \qquad \text{for real values of } x, \quad x \neq -0.5$$

(a) Explain why f does not have an inverse. *(1 mark)*

(b) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

(c) State the range of g^{-1}. *(1 mark)*

(d) Solve the equation $fg(x) = g(x)$. *(3 marks)*

6 (a) Given that $3\ln x = 4$, find the exact value of x. *(1 mark)*

(b) By forming a quadratic equation in $\ln x$, solve $3\ln x + \dfrac{20}{\ln x} = 19$, giving your answers for x in an exact form. *(5 marks)*

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7 (a) On separate diagrams:

(i) sketch the curve with equation $y = |3x + 3|$; *(2 marks)*

(ii) sketch the curve with equation $y = |x^2 - 1|$. *(3 marks)*

(b) (i) Solve the equation $|3x + 3| = |x^2 - 1|$. *(5 marks)*

(ii) Hence solve the inequality $|3x + 3| < |x^2 - 1|$. *(2 marks)*

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8 Use the substitution $u = 1 + 2\tan x$ to find

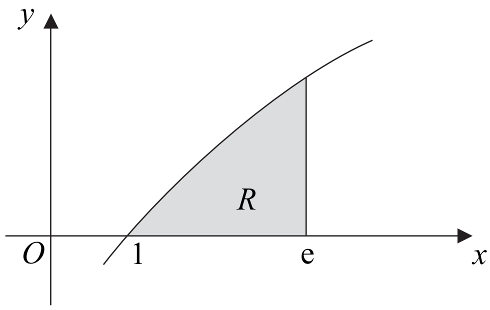
$$\int \frac{1}{(1 + 2\tan x)^2 \cos^2 x}\,\mathrm{d}x$$

(5 marks)

9 (a) Use integration by parts to find $\int x \ln x \, dx$. *(3 marks)*

(b) Given that $y = (\ln x)^2$, find $\dfrac{dy}{dx}$. *(2 marks)*

(c) The diagram shows part of the curve with equation $y = \sqrt{x} \ln x$.



The shaded region R is bounded by the curve $y = \sqrt{x} \ln x$, the line $x = \mathrm{e}$ and the x-axis from $x = 1$ to $x = \mathrm{e}$.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in an exact form. *(6 marks)*

END OF QUESTIONS